Geometric view of eigenvalue and eigenvectors

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RECENT ADVANCES AND APPLICATIONS IN MATHEMATICS
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Department of Mathematics
University College of Science
Osmania University

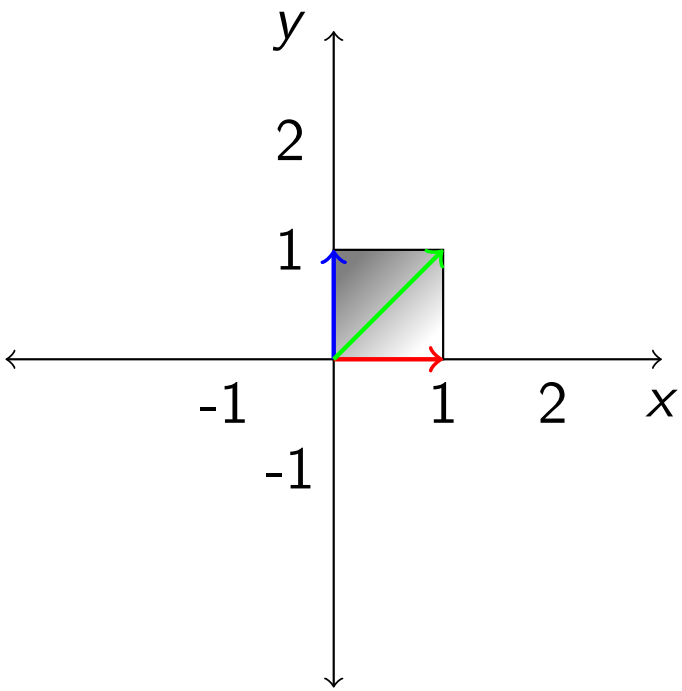
Maps which preserve

- Origin
- lines passing through origin
- parallelograms with one corner as origin

Outline

1. Linear transformations on plane
   - Typical Examples
   - Properties
   - Eigen value and eigen vector
   - Problem
2. Markov Matrices
   - Formation
   - Interpretation
   - Properties
3. Approximate eigenvalue

Rotation $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$



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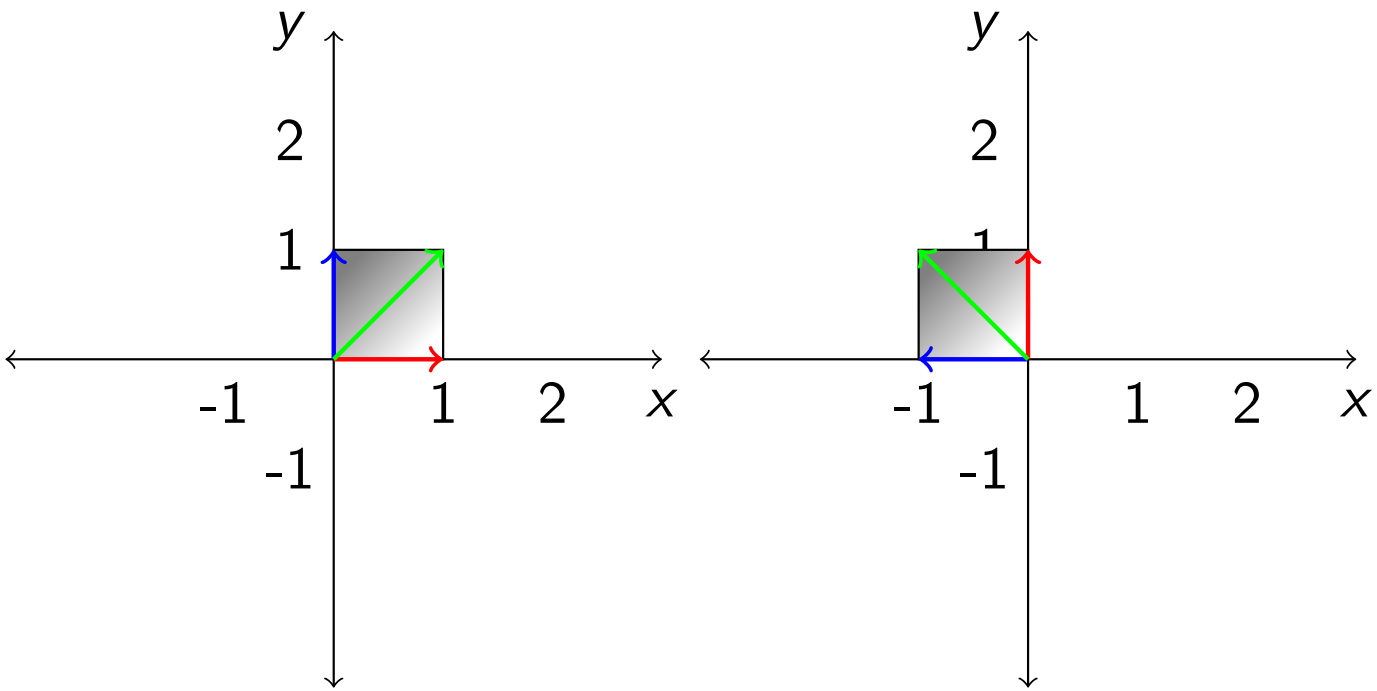
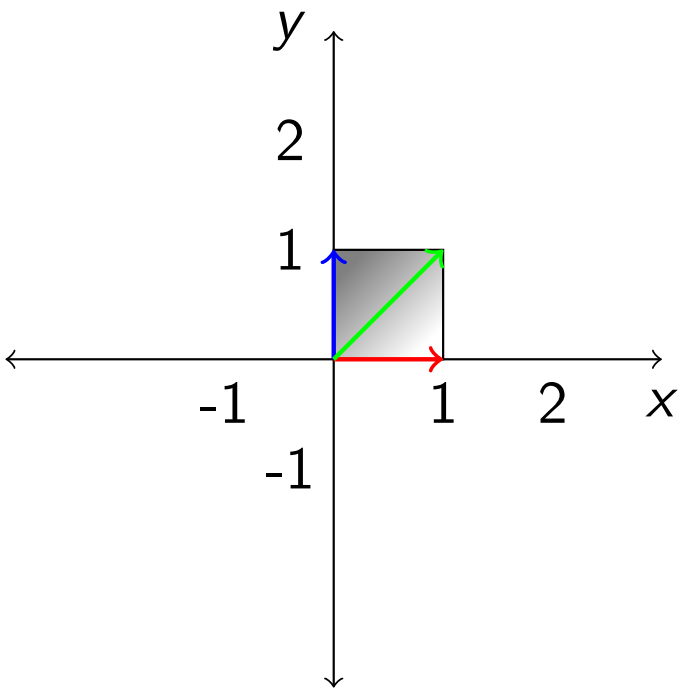


Reflection $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$



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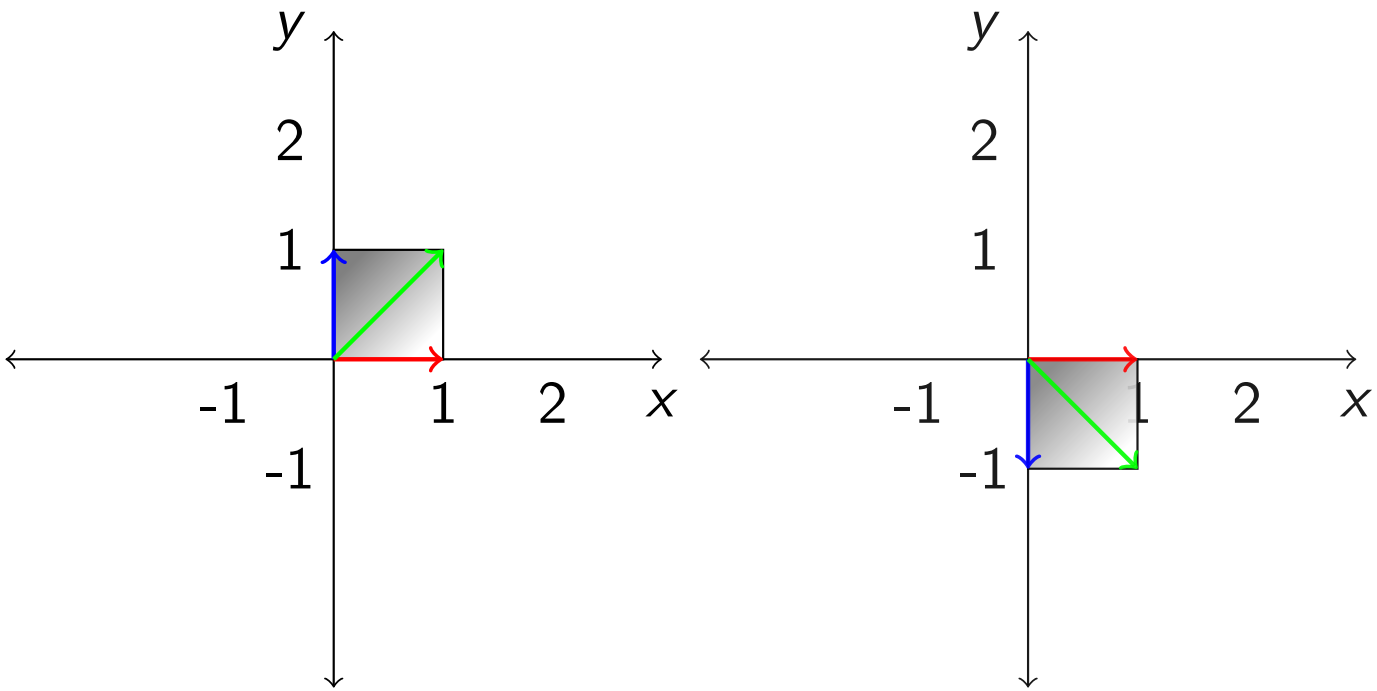
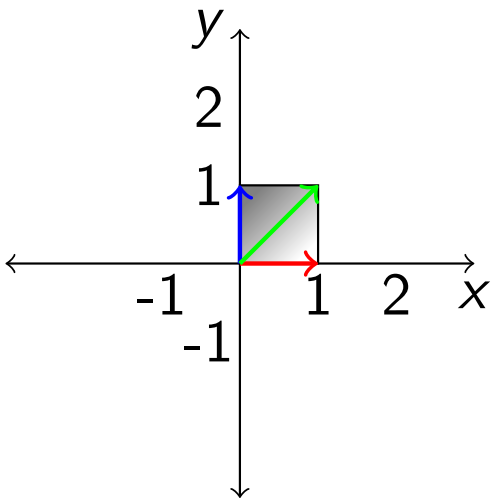


Expansion $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ Compression $\begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$



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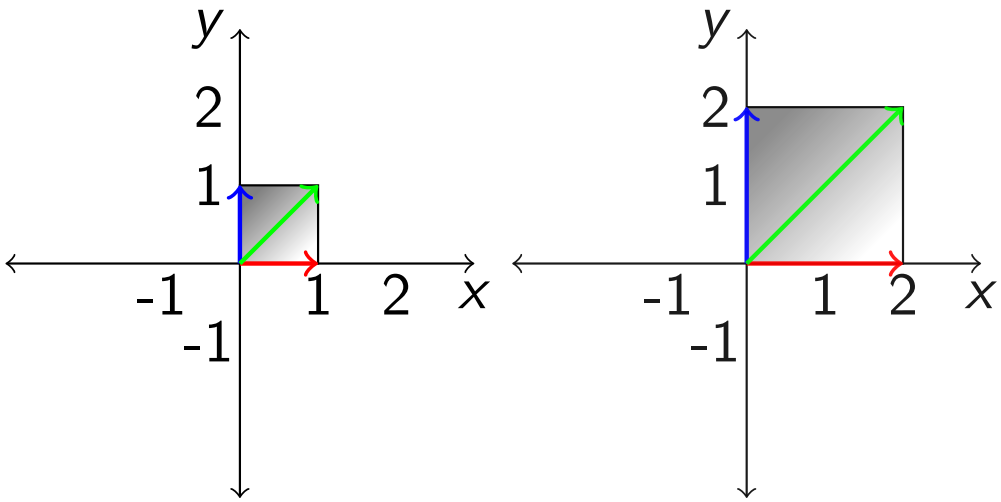

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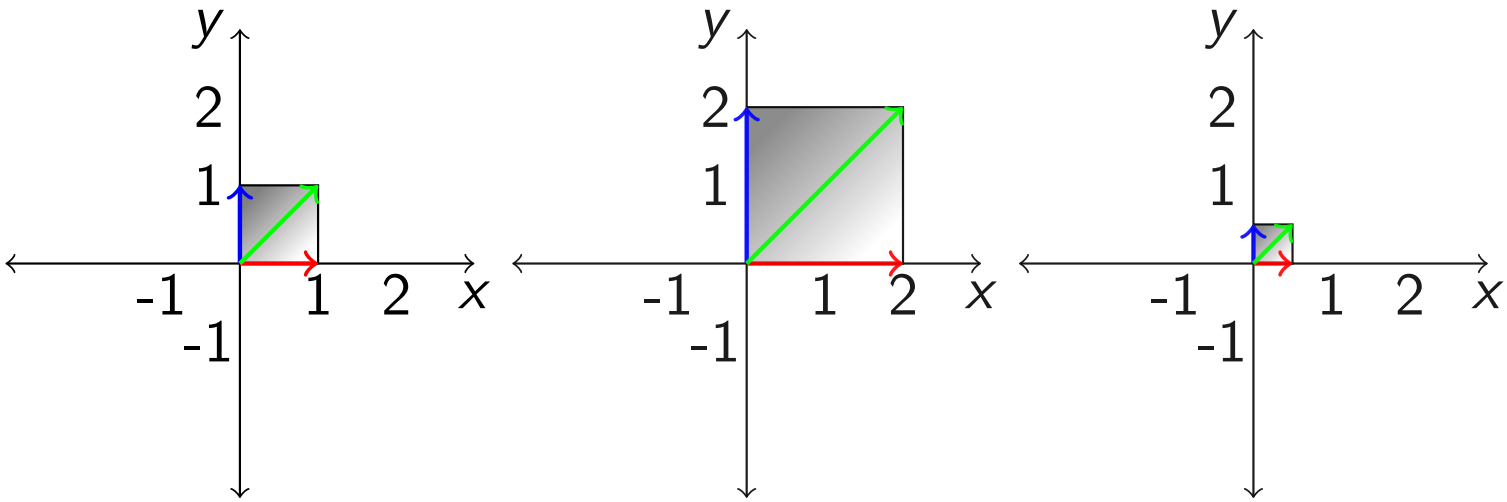
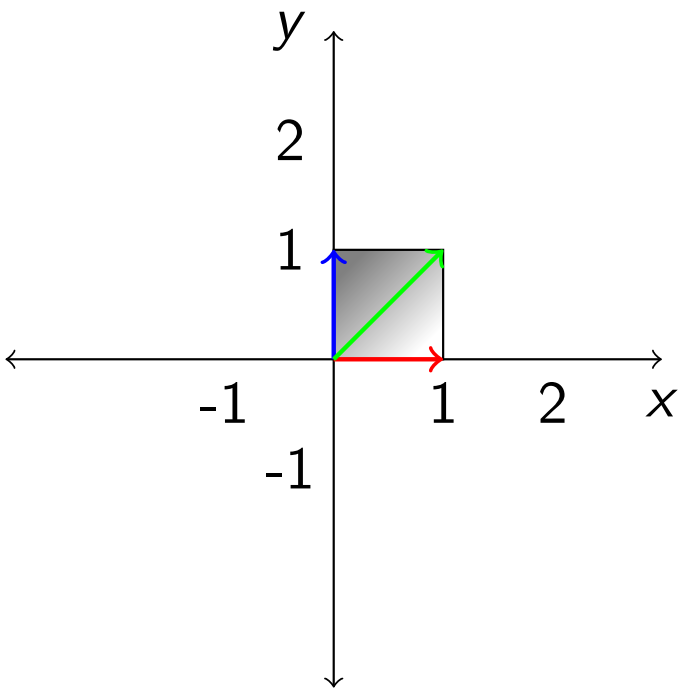


Multi-scaling or Stretching $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$



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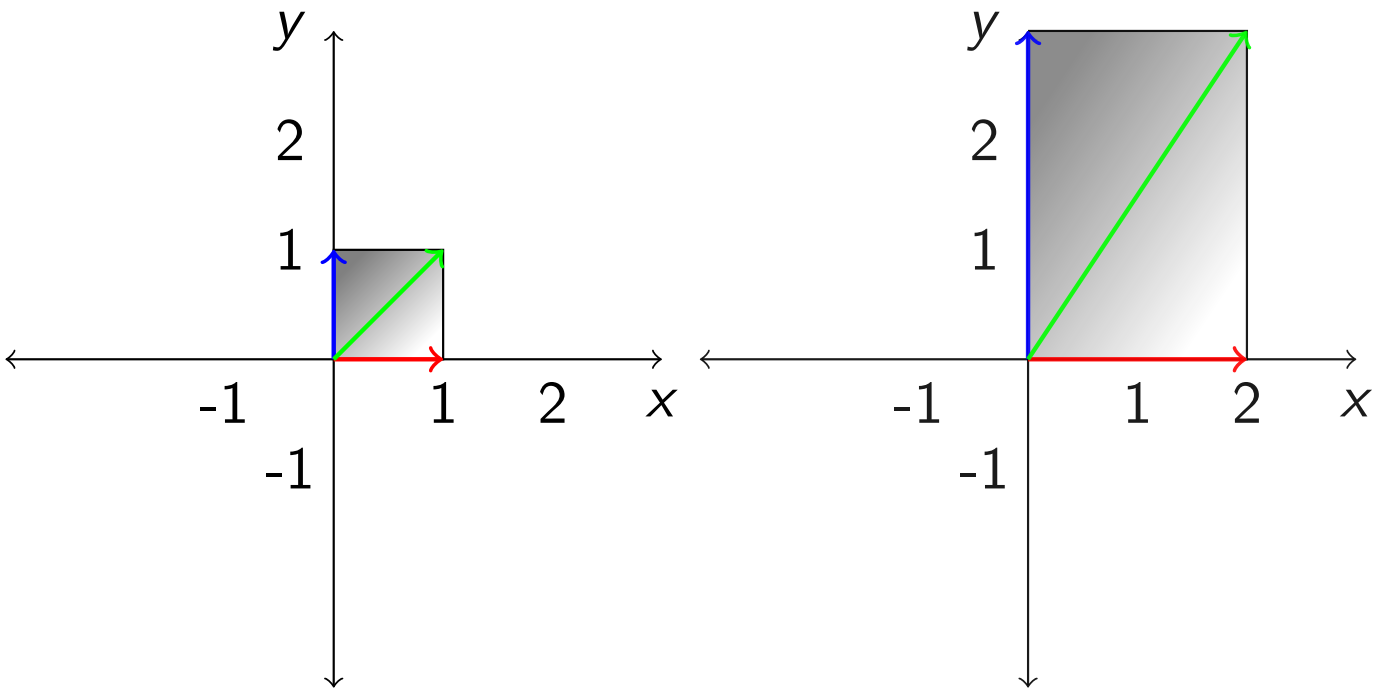

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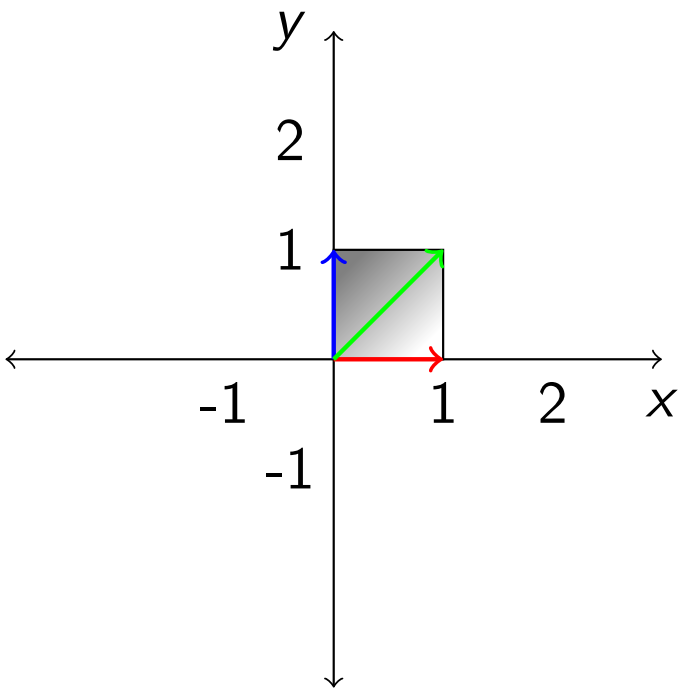

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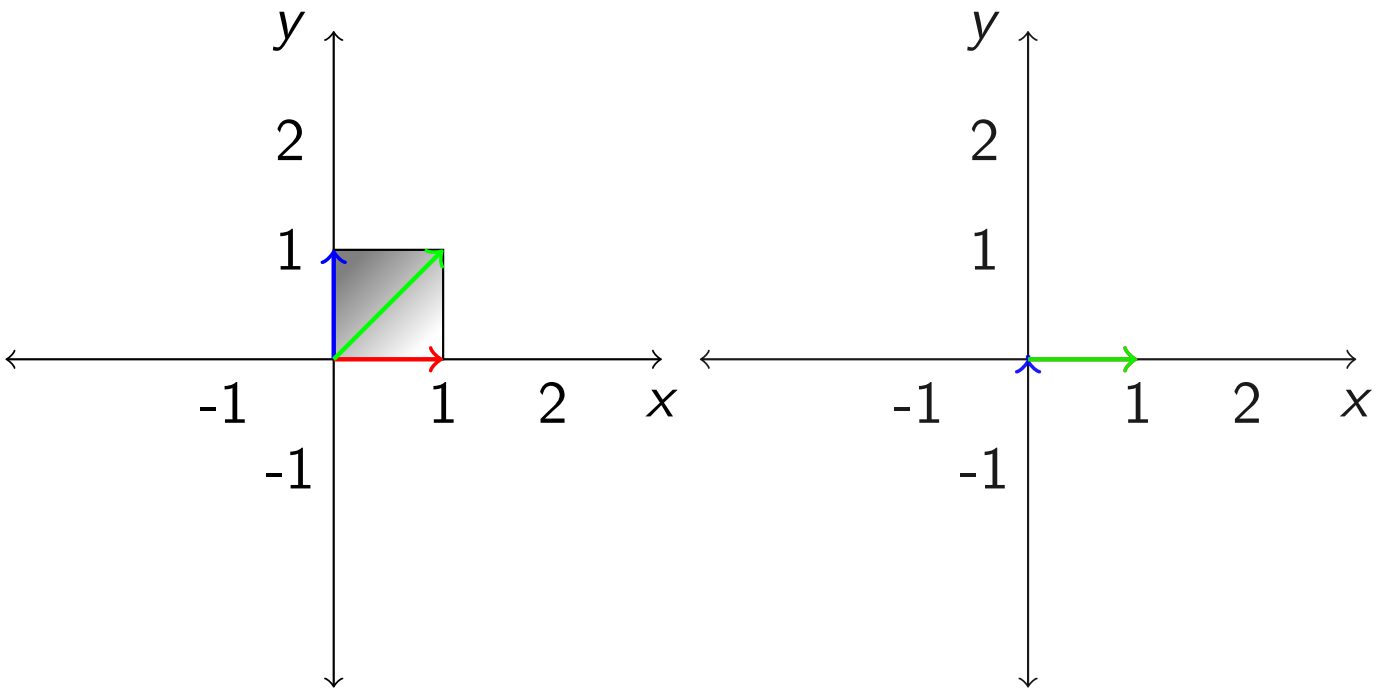

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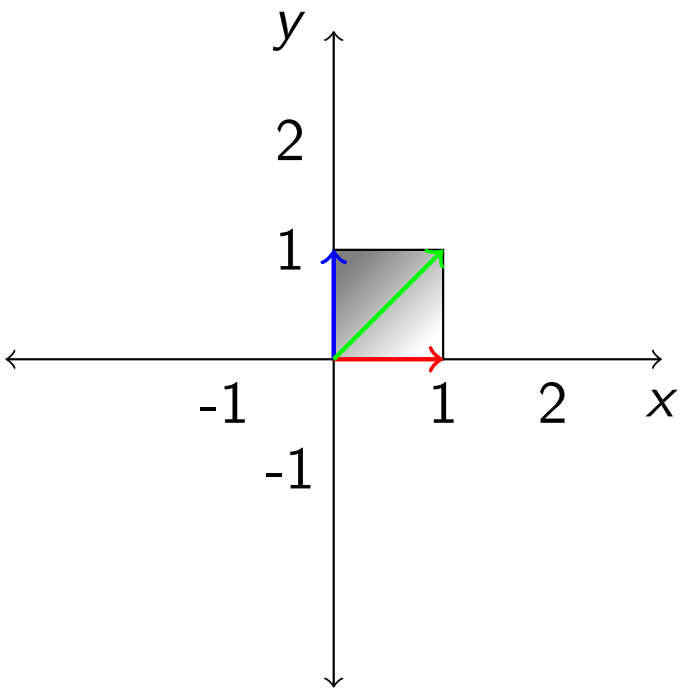

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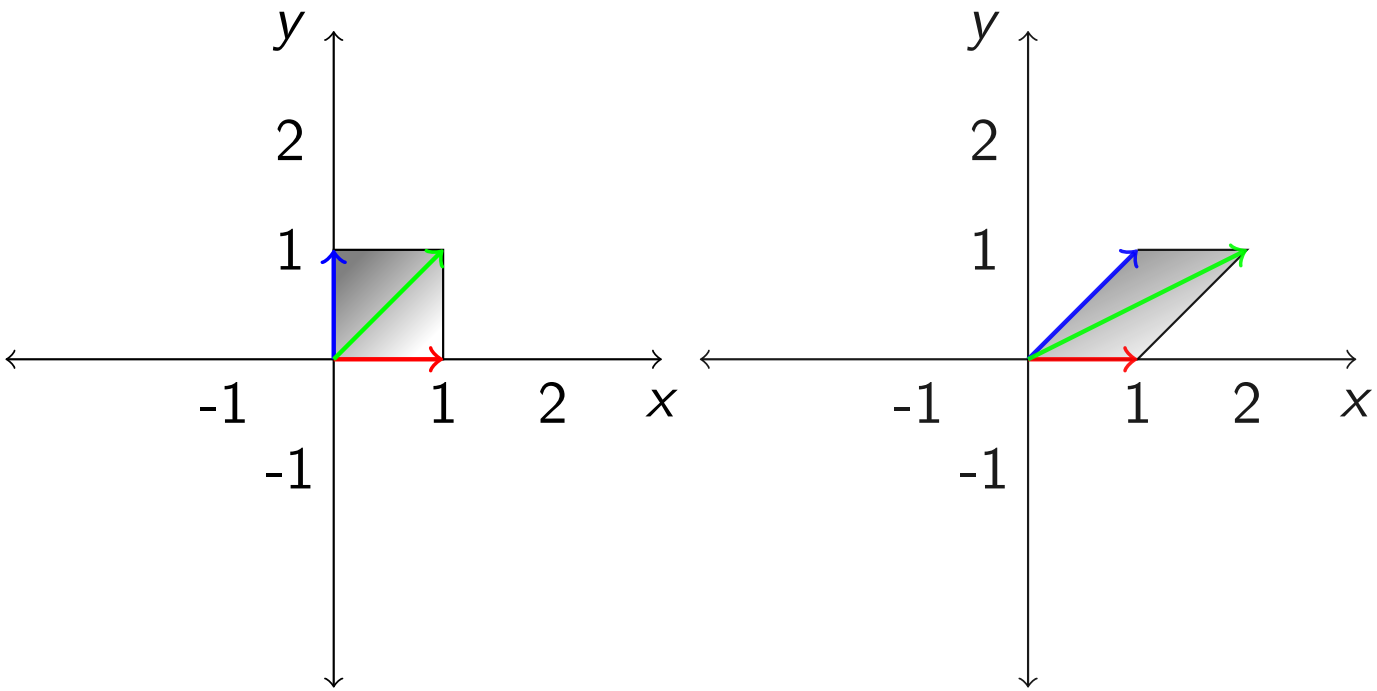

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   - Typical Examples
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   - Eigen value and eigen vector
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2. Markov Matrices
   - Formation
   - Interpretation
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3. Approximate eigenvalue

Properties

- Area

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- Area
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Table of properties

Map	Area	Fixed Dir Eigenvector	Scale in FD Eigenvalue	Det	Diagonable
Rotation	1	NO	NO	1	NO
Reflection	1	x-axis, y-axis	1,-1	-1	Yes
Expansion	4	x-axis, y-axis	2, 2	4	Yes
Compression	1/4	x-axis, y-axis	1/2,1/2	1/4	Yes
Multi-scaling	6	x-axis, y-axis	2,3	6	Yes
Projection	0	x-axis, y-axis	1,0	0	Yes
Shear	1	x-axis	1	1	NO

Table: Properties

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Let A a be square matrix.

- Eigen values of A are solutions or roots of

$$\det(A - \lambda I) = 0.$$

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  - x is an eigenvector corresponding to the eigenvalue λ.

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Problem

Big Problem

- Getting a common opinion from individual opinion

Purpose

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- Showing all steps of this process using linear algebra

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Purpose

- Showing all steps of this process using linear algebra
- Mainly using eigenvalues and eigenvectors

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   Total mark: 100+20+50+30+0=200

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3. Arranging: Writing them in a matrix form

$$\begin{pmatrix} 0.5 & 0.8 & 0.0 & 0.4 & 0.2 \\ 0.1 & 0.32 & 0.0 & 0.0 & 0.2 \\ 0.25 & 0.24 & 1.0 & 0.0 & 0.2 \\ 0.15 & 0.16 & 0.0 & 0.4 & 0.2 \\ 0.0 & 0.2 & 0.0 & 0.2 & 0.2 \end{pmatrix}$$

	Isha	*Jose*	*Kumar*	*Latha*	*Mani*
Isha	0.5	0.8	0.0	0.4	0.2
Jose	0.1	0.32	0.0	0.0	0.2
Kumar	0.25	0.24	1.0	0.0	0.2
Latha	0.15	0.16	0.0	0.4	0.2
Mani	0.0	0.2	0.0	0.2	0.2
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
	sum is 1	1	1	1	1

Markov Matrix

$$M = \begin{bmatrix} m_{11} & m_{12} & \cdots & m_{1n} \\ m_{21} & m_{22} & & m_{2n} \\ \vdots & \vdots & & \vdots \\ m_{n1} & m_{n2} & & m_{nn} \end{bmatrix}_{n \times n}$$

Definition (Markov Matrix)

A real $n \times n$ matrix is called a Markov matrix if

1. Each entry is non-negative: $m_{ij} \geq 0$ for all $1 \leq i \leq n, 1 \leq j \leq n$.

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2. Each column sum is 1: $\sum_{i=1}^{n} m_{ij} = 1$ for each $1 \leq j \leq n$.

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Interpretation

- We have formed a Markov matrix from individual opinions.

$$\begin{pmatrix} 0.5 & 0.8 & 0.0 & 0.4 & 0.2 \\ 0.1 & 0.32 & 0.0 & 0.0 & 0.2 \\ 0.25 & 0.24 & 1.0 & 0.0 & 0.2 \\ 0.15 & 0.16 & 0.0 & 0.4 & 0.2 \\ 0.0 & 0.2 & 0.0 & 0.2 & 0.2 \end{pmatrix}$$

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- We have formed a Markov matrix from individual opinions.
- If Isha's opinion is full value then Isha is better than others.
- If Jose's opinion is full value then Isha is better than others.
- If Mani's opinion is full value then all are equal.

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- When the opinion values matches with the conclusion value?. For which opinion value we will get the same conclusion value.

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- For M, find an eigenvector x corresponding to eigenvalue 1.

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For a Markov matrix M every column sum is 1. Consider the transpose matrix M^T. Now its row sum is 1. Let $e = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$

- $M^T e = e$

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- But eigen vectors of A and A^T need not be same.

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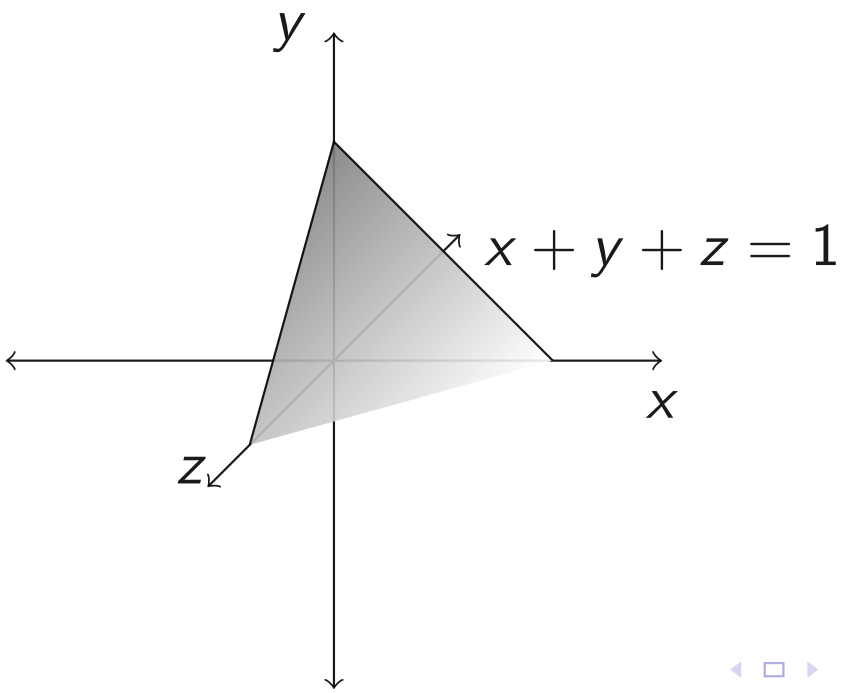
Theorem (Brouwer fixed-point theorem)

Every continuous function from a convex compact subset K of a Euclidean space to K itself has a fixed point



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3. *There is an eigenvector corresponding to the eigenvalue 1.*
4. *This eigenvector has all positive entries with sum 1.*
5. *Such an eigenvector is unique.*

Finding the stable opinion

Now we know that every Markov matrix with positive entries has a unique eigenvector corresponding to the eigenvalue $\lambda = 1$.

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$$M - \lambda I = \begin{bmatrix} m_{11} & m_{12} & \cdots & m_{1n} \\ m_{21} & m_{22} & & m_{2n} \\ \vdots & \vdots & & \vdots \\ m_{n1} & m_{n2} & & m_{nn} \end{bmatrix}_{n\times n} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 1 \end{bmatrix}_{n\times n}$$

Finding the stable opinion

Now we know that every Markov matrix with positive entries has a unique eigenvector corresponding to the eigenvalue $\lambda = 1$.

$$M - \lambda I = \begin{bmatrix} m_{11} & m_{12} & \cdots & m_{1n} \\ m_{21} & m_{22} & & m_{2n} \\ \vdots & \vdots & & \vdots \\ m_{n1} & m_{n2} & & m_{nn} \end{bmatrix}_{n\times n} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 1 \end{bmatrix}_{n\times n}$$

$$(M - \lambda I)x = \begin{bmatrix} m_{11} - \lambda & m_{12} & \cdots & m_{1n} \\ m_{21} & m_{22} - \lambda & & m_{2n} \\ \vdots & \vdots & & \vdots \\ m_{n1} & m_{n2} & & m_{nn} - \lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

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Solve $(M - \lambda I)x = 0$ by Gauss Elimination Process

Other Views

- It is a simpler version of page-rank.

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Approximate eigenvalue

Definition (Pseudospectrum)

λ is an ϵ-pseudo-eigenvalue of M if $\|(M-\lambda)^{-1}\| \geq \epsilon$

Definition (Condition-spectrum)

λ is an ϵ-condition-eigenvalue of M if $\|(M-\lambda)\|\|(M-\lambda)^{-1}\| \geq \epsilon$

Questions

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Thank you

You know better

In aeronautical engineering eigenvalues may determine whether the flow over a wing is laminar or turbulent.

In electrical engineering they may determine the frequency response of an amplifier or the reliability of a national power system.

In structural mechanics eigenvalues may determine whether an automobile is too noisy or whether a building will collapse in an earth-quake.

In probability they may determine the rate of convergence of a Markov process.

In ecology they may determine whether a food web will settle into a steady equilibrium.

In numerical analysis they may determine whether a discretization of a differential equation will get the right answer or how fast a conjugate gradient iteration will converge.